

$B_c \rightarrow B_{sJ}$ form factors and B_c decays into B_{sJ} in covariant light-front approach

Yu-Ji Shi^{1,a}, Wei Wang^{1,2,b}, Zhen-Xing Zhao^{1,c}

¹ INPAC, Shanghai Key Laboratory for Particle Physics and Cosmology, Department of Physics and Astronomy, Shanghai Jiao-Tong University, Shanghai 200240, China

² State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

Received: 6 July 2016 / Accepted: 26 September 2016 / Published online: 12 October 2016
© The Author(s) 2016. This article is published with open access at Springerlink.com

Abstract We suggest to study the B_s and its excitations B_{sJ} in the B_c decays. We calculate the $B_c \rightarrow B_{sJ}$ and $B_c \rightarrow B_J$ form factors within the covariant light-front quark model, where the B_{sJ} and B_J denote an s-wave or p-wave $\bar{b}s$ and $\bar{b}d$ meson, respectively. The form factors at $q^2 = 0$ are directly computed while their q^2 -distributions are obtained by extrapolation. The derived form factors are then used to study semileptonic $B_c \rightarrow (B_{sJ}, B_J)\bar{\ell}\nu$ decays, and non-leptonic $B_c \rightarrow B_{sJ}\pi$. Branching fractions and polarizations are predicted in the standard model. We find that the branching fractions are sizable and might be accessible at the LHC experiment and future high-energy e^+e^- colliders with a high luminosity at the Z-pole. The future experimental measurements are helpful to study the nonperturbative QCD dynamics in the presence of a heavy spectator and also of great value for the study of spectroscopy.

1 Introduction

In the past decades, there has been a lot of progress in hadron spectroscopy, thanks to the well-operating experiments including the e^+e^- colliders and hadron colliders. The immense interest in spectroscopy is not only due to the fact that one is able to find many missing hadrons to complete the quark model, but more importantly due to the observations of states that are unexpected in the simple quark model. The latter ones are generally called hadron exotics. A milestone in the exploration of the exotics is the discovery of $X(3872)$, first in B decays by Belle Collaboration [1] and subsequently confirmed in many distinct processes in different experiments [2–4]. It was found the prop-

erties of this meson are peculiar. Since then the identification of multiquark hadrons has become a focus topic in hadron physics. Inspired by the discovery of $X(3872)$, a number of new interesting structures were discovered in the mass region of heavy quarkonium. Refer to Refs. [5–8] for recent reviews.

On the theoretical side, deciphering the underlying dynamics of these multiquark states is a formidable challenge, and is often based on explicit and distinct assumptions. In many assumptions, the quarkonium-like states are usually composed of a pair of heavy constituents, which makes it vital to study first the heavy–light hadron. In the system with one heavy charm quark, a series of important results have started with the discoveries of the narrow states $D_s(2317)$ in the $D_s^+\pi^0$ final state and $D_s(2460)$ in the $D_s^*\pi^0$ and $D_s\gamma$ final state [9, 10]. Along this line, a few other new states, such as $D_{s1}(2536)$, $D_{s2}(2573)$, $D_s(2710)$, have been observed at the B factory and other facilities [11].

Bottomed hadrons are related to charmed mesons by heavy quark symmetry. But compared to the charm sector, there is less progress for the bottomed hadrons. In experiment, only a few bottom–strange mesons are observed, most of which are believed to be in agreement with the quark model. In this paper, we propose to use the B_c decays and study the spectrum of the B_{sJ} . We observe a few advantages. First of all, the large production rate of the B_c is in expectation, in particular the LHCb will produce a number of B_c events and thus the $B_c \rightarrow B_{sJ}$ decays will have a large potential to be observed. Second, the scale over the m_W can be computed in the perturbation theory and the QCD evolution between the m_W and the low energy scale m_c is well organized by making use of the renormalization group improved perturbation theory. Consequently, the B_c decays into B_s and other excited states have received some theoretical attention [12–30]. In the following we will address the investigation of the production rates of B_{sJ} meson (an s-wave or p-wave $\bar{b}s$ hadron) in semileptonic

^a e-mail: shiyuji@sjtu.edu.cn

^b e-mail: wei.wang@sjtu.edu.cn

^c e-mail: star_0027@sjtu.edu.cn

and nonleptonic B_c meson decays in the framework of the covariant light-front quark model (LFQM) [31].

In the $B_c \rightarrow B_{sJ}$ decays, the quark level transition is $c \rightarrow s$ in which the heavy bottom quark acts as a spectator. Since most of the momentum of the hadron is carried by the spectator, there is no large momentum transfer and the transition is dominated by the soft mechanism. A form factor can then be expressed as an overlap of the wave functions of the initial and final state hadrons. Treatments in quark models like the LFQM are of this type.

As pointed out in Ref. [32], the light-front approach shows some unique features which are suitable to handle a hadronic bound state. The LFQM [33–36] can provide a relativistic treatment of moving hadrons and give a full treatment of hadron spins in terms of the Melosh rotation. Light-front wave functions, which characterize the hadron in terms of their fundamental quark and gluon degrees of freedom, are independent of hadron momentum and thus are Lorentz invariant. Moreover, in covariant LFQM [31], the spurious contribution which depends on the orientation of the light front is elegantly eliminated by including zero-mode contributions. This covariant model has been successfully extended to study the decay constants and form factors of various mesons [37–49]. Through this study of $B_c \rightarrow B_{sJ}$ in LFQM, we believe that one will not only obtain the information as regards the decay dynamics in the presence of a heavy spectator but will also provide a side-check for the classification of the heavy–light mesons. It is also helpful toward the establishment of a global picture of the heavy–light spectroscopy including the exotic spectrum.

The rest of this paper is organized as follows. In Sect. 2, we will give a brief description of the parametrization of form factors, the framework of covariant LFQM, and the form factor calculation in this model. We present our numerical results for various transitions in Sect. 3. In Sect. 4, we use the form factors to study semileptonic and nonleptonic B_c decays. In this section, we will present our predictions for branching fractions and polarizations. The last section contains a brief summary.

2 Transition form factors in the covariant LFQM

2.1 $B_c \rightarrow B_{sJ}$ form factors

The effective electroweak Hamiltonian for the $B_c \rightarrow B_{sJ} \bar{l} \nu$ reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* [\bar{s} \gamma_\mu (1 - \gamma_5) c] [\bar{\nu} \gamma^\mu (1 - \gamma_5) l], \quad (1)$$

where G_F and V_{cs} are the Fermi constant and Cabibbo–Kobayashi–Maskawa matrix element, respectively. Leptonic

parts can be computed in perturbation theory while hadronic contributions are parametrized in terms of form factors.

An s -wave meson corresponds to a pseudoscalar meson or a vector meson, abbreviated as P and V , respectively. For a p -wave meson, the involved state is a scalar S , an axial-vector A or a tensor meson T . In the following we introduce the abbreviations $P = P' + P''$, $q = P' - P''$, and we adopt the convention of $\epsilon_{0123} = 1$. The $B_c \rightarrow P, V$ form factors can be defined as follows:

$$\begin{aligned} \langle P(P'') | V_\mu | B_c(P') \rangle &= \left(P_\mu - \frac{m_{B_c}^2 - m_P^2}{q^2} q_\mu \right) F_1^{B_c P}(q^2) \\ &\quad + \frac{m_{B_c}^2 - m_P^2}{q^2} q_\mu F_0^{B_c P}(q^2), \\ \langle V(P'', \varepsilon'') | V_\mu | B_c(P') \rangle &= -\frac{1}{m_{B_c} + m_V} \\ &\quad \times \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta V^{B_c V}(q^2), \\ \langle V(P'', \varepsilon'') | A_\mu | B_c(P') \rangle &= 2im_V \frac{\varepsilon''^* \cdot q}{q^2} q_\mu A_0^{B_c V}(q^2) \\ &\quad + i(m_{B_c} + m_V) A_1^{B_c V}(q^2) \left[\varepsilon''^* - \frac{\varepsilon''^* \cdot q}{q^2} q_\mu \right] \\ &\quad - i \frac{\varepsilon''^* \cdot P}{m_{B_c} + m_V} A_2^{B_c V}(q^2) \left[P_\mu - \frac{m_B^2 - m_V^2}{q^2} q_\mu \right]. \end{aligned} \quad (2)$$

In analogy with $B_c \rightarrow V$ form factors, we parametrize the $B_c \rightarrow T$ form factors as

$$\begin{aligned} \langle T(P'', \varepsilon'') | V_\mu | B_c(P') \rangle &= -\frac{2V^{B_c T}(q^2)}{m_{B_c} + m_T} \epsilon^{\mu\nu\rho\sigma} (\varepsilon_T^*)_v (P')_\rho (P'')_\sigma, \\ \langle T(P'', \varepsilon'') | A_\mu | B_c(P') \rangle &= 2im_T \frac{\varepsilon_T^* \cdot q}{q^2} q_\mu A_0^{B_c T}(q^2) \\ &\quad + i(m_{B_c} + m_T) A_1^{B_c T}(q^2) \left[\varepsilon_T^* - \frac{\varepsilon_T^* \cdot q}{q^2} q_\mu \right] \\ &\quad - i \frac{\varepsilon_T^* \cdot q}{m_{B_c} + m_T} A_2^{B_c T}(q^2) \left[P_\mu - \frac{m_{B_c}^2 - m_T^2}{q^2} q_\mu \right], \end{aligned} \quad (3)$$

with

$$\varepsilon_{T\mu}(h) = \frac{1}{m_{B_c}} \varepsilon''_{\mu\nu}(h) P'^\nu. \quad (4)$$

The $B_c \rightarrow S, A$ form factors can be defined by exchanging the vector and axial-vector current:

$$\begin{aligned} \langle S(P'') | A_\mu | B_c(P') \rangle &= -i \left[\left(P_\mu - \frac{m_{B_c}^2 - m_S^2}{q^2} q_\mu \right) \right. \\ &\quad \times F_1^{B_c S}(q^2) + \left. \frac{m_{B_c}^2 - m_S^2}{q^2} q_\mu F_0^{B_c S}(q^2) \right], \\ \langle A(P'', \varepsilon'') | V_\mu | B_c(P') \rangle &= -2m_A \frac{\varepsilon''^* \cdot q}{q^2} q_\mu V_0^{B_c A}(q^2) \end{aligned}$$

$$\begin{aligned}
& -(m_{B_c} + m_A) V_1^{B_c A}(q^2) \left[\varepsilon_{\mu}^{\prime\prime*} - \frac{\varepsilon^{\prime\prime*} \cdot q}{q^2} q_{\mu} \right] \\
& + \frac{\varepsilon^{\prime\prime*} \cdot P}{m_{B_c} + m_A} V_2^{B_c A}(q^2) \left[P_{\mu} - \frac{m_B^2 - m_A^2}{q^2} q_{\mu} \right], \\
\langle A(P'', \varepsilon'') | A_{\mu} | B_c(P') \rangle &= -i \frac{1}{m_{B_c} - m_A} \\
& \times \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\prime\prime* \nu} P^{\alpha} q^{\beta} A^{B_c A}(q^2). \quad (5)
\end{aligned}$$

The spin-2 polarization tensor can be constructed using the standard polarization vector ε :

$$\begin{aligned}
\varepsilon_{\mu\nu}^{\prime\prime}(P'', \pm 2) &= \varepsilon_{\mu}(\pm) \varepsilon_{\nu}(\pm), \\
\varepsilon_{\mu\nu}^{\prime\prime}(P'', \pm 1) &= \frac{1}{\sqrt{2}} [\varepsilon_{\mu}(\pm) \varepsilon_{\nu}(0) + \varepsilon_{\nu}(\pm) \varepsilon_{\mu}(0)], \\
\varepsilon_{\mu\nu}^{\prime\prime}(P'', 0) &= \frac{1}{\sqrt{6}} [\varepsilon_{\mu}(+) \varepsilon_{\nu}(-) + \varepsilon_{\nu}(+) \varepsilon_{\mu}(-)] \\
&+ \sqrt{\frac{2}{3}} \varepsilon_{\mu}(0) \varepsilon_{\nu}(0). \quad (6)
\end{aligned}$$

It is symmetric and traceless, and $\varepsilon_{\mu\nu}^{\prime\prime} P^{\prime\prime \nu} = 0$. If the recoiling meson is moving on the plus direction of the z axis, their explicit structures are chosen as

$$\varepsilon_{\mu}(0) = \frac{1}{m_T} (|\vec{p}_T|, 0, 0, -E_T), \quad \varepsilon_{\mu}(\pm) = \frac{1}{\sqrt{2}} (0, \pm 1, i, 0), \quad (7)$$

where E_T and $|\vec{p}_T|$ are the energy and the momentum magnitude of the tensor meson in the B_c rest frame, respectively.

2.2 Covariant light-front approach

In the covariant LFQM, it is convenient to use the light-front decomposition of the momentum $P' = (P'^-, P'^+, P'_{\perp})$, with $P'^{\pm} = P'^0 \pm P'^3$, and thus $P'^2 = P'^+ P'^- - P'_{\perp}^2$. The incoming (outgoing) meson has the momentum $P' = p'_1 + p_2$ ($P'' = p'_1 + p_2$) and the mass M' (M''). The quark and antiquark inside the incoming (outgoing) meson have the mass $m_1^{(\prime\prime)}$ and m_2 , respectively. Their momenta are denoted as $p_1^{(\prime\prime)}$ and p_2 respectively. In particular these momenta can be written in terms of the internal variables (x_i, p'_{\perp}) by

$$p_{1,2}^{\prime+} = x_{1,2} P^{\prime+}, \quad p_{1,2\perp}^{\prime} = x_{1,2} P'_{\perp} \pm p'_{\perp}, \quad (8)$$

with the momentum fractions $x_1 + x_2 = 1$. With these internal variables, one can define some useful quantities for both incoming and outgoing mesons:

$$\begin{aligned}
M_0^{\prime 2} &= (e'_1 + e_2)^2 = \frac{p_{\perp}^2 + m_1^{\prime 2}}{x_1} + \frac{p_{\perp}^2 + m_2^2}{x_2}, \\
\tilde{M}'_0 &= \sqrt{M_0^{\prime 2} - (m'_1 - m_2)^2}, \\
e_i^{(\prime)} &= \sqrt{m_i^{(\prime)2} + p_{\perp}^2 + p_z^2}, \quad p_z' = \frac{x_2 M_0'}{2} - \frac{m_2^2 + p_{\perp}^2}{2x_2 M_0'}. \quad (9)
\end{aligned}$$

Table 1 Meson–quark–antiquark vertices used in the covariant LFQM. In the case of the outgoing meson, one should use instead $i(\gamma_0 \Gamma_M^{\dagger} \gamma_0)$ for the corresponding vertices

$M(^{2S+1}L_J)$	$i \Gamma_M'$
Pseudoscalar (1S_0)	$H'_p \gamma_5$
Scalar (3P_0)	H'_S
Vector (3S_1)	$i H'_V [\gamma_{\mu} - \frac{1}{W'_V} (p'_1 - p_2)_{\mu}]$
Axial (3P_1)	$i H'_{3A} [\gamma_{\mu} + \frac{1}{W'_{3A}} (p'_1 - p_2)_{\mu}] \gamma_5$
Axial (1P_1)	$i H'_{1A} [\frac{1}{W'_{1A}} (p'_1 - p_2)_{\mu}] \gamma_5$
Tensor (3P_2)	$i \frac{1}{2} H'_T [\gamma_{\mu} - \frac{1}{W'_T} (p'_1 - p_2)_{\mu}] (p'_1 - p_2)_{\nu}$

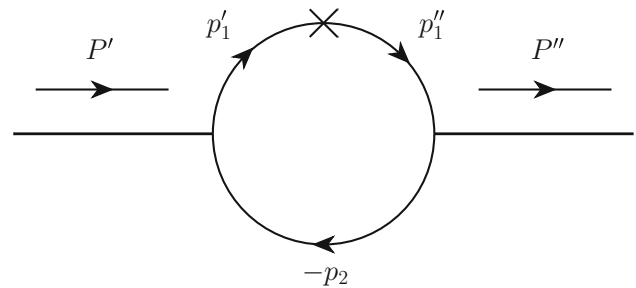


Fig. 1 Feynman diagram for transition form factors, where the cross symbol in the diagram denotes the transition current

The Feynman rules for meson–quark–antiquark vertices can be derived using the conventional light-front approach, whose forms for the s-wave and p-wave states are collected in Table 1 [31,37]. An extension to the d-wave vertices has been conducted in Ref. [50]. In the following we will take the $B_c \rightarrow B_s$ transition as the example and illustrate the calculation. To do so, we will consider the matrix element

$$\langle P(P'') | V_{\mu} | P(P') \rangle \equiv \mathcal{B}_{\mu}^{PP}, \quad (10)$$

whose Feynman diagram is shown in Fig. 1. It is straightforward to obtain

$$\mathcal{B}_{\mu}^{PP} = i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{H'_P H''_P}{N'_1 N''_1 N_2} S_{V\mu}^{PP}, \quad (11)$$

where

$$\begin{aligned}
S_{V\mu}^{PP} &= \text{Tr}[\gamma_5 (\not{p}'_1 + m_1') \gamma_{\mu} (\not{p}'_1 + m'_1) \gamma_5 (\not{p}_2 - m_2)], \\
N_1^{(\prime\prime)} &= p_1^{(\prime\prime)2} - m_1^{(\prime\prime)2} + i\epsilon, \quad N_2 = p_2^2 - m_2^2 + i\epsilon. \quad (12)
\end{aligned}$$

Here we consider the $q^+ = 0$ frame. The $p_1^{\prime-}$ integration picks up the pole $p_2 = \hat{p}_2 = [(p_{2\perp}^2 + m_2^2)/p_2^+, p_2^+, p_{2\perp}]$ and leads to

$$\begin{aligned}
N_1^{(\prime\prime)} &\rightarrow \hat{N}_1^{(\prime\prime)} = x_1(M^{\prime\prime 2} - M_0^{\prime\prime 2}), \\
H_P^{(\prime\prime)} &\rightarrow h_P^{(\prime\prime)}, \\
\int \frac{d^4 p'_1}{N_1' N_1'' N_2} H_P' H_P'' S^{PP} &\rightarrow -i\pi \int \frac{dx_2 d^2 p'_\perp}{x_2 \hat{N}_1' \hat{N}_1''} h_P' h_P'' \hat{S}^{PP},
\end{aligned} \quad (13)$$

where

$$M_0^{\prime\prime 2} = \frac{p_\perp^{\prime\prime 2} + m_1^{\prime\prime 2}}{x_1} + \frac{p_\perp^{\prime\prime 2} + m_2^2}{x_2}, \quad (14)$$

with $p_\perp^{\prime\prime} = p'_\perp - x_2 q_\perp$. The explicit form of h_P' has been derived in Refs. [31, 37]

$$h_P' = (M^{\prime 2} - M_0^{\prime 2}) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}_0'} \varphi', \quad (15)$$

where φ' is the light-front momentum distribution amplitude for s -wave meson. In practice, the following Gaussian-type wave function can be adopted [31, 37]:

$$\varphi' = \varphi'(x_2, p'_\perp) = 4 \left(\frac{\pi}{\beta'^2} \right)^{3/4} \sqrt{\frac{dp'_z}{dx_2}} \exp \left(-\frac{p_z'^2 + p_\perp'^2}{2\beta'^2} \right). \quad (16)$$

As shown in Refs. [31, 37], the inclusion of the so-called zero-mode contribution in the above matrix elements in practice amounts to the replacements

$$\begin{aligned}
\hat{p}_{1\mu} &\doteq P_\mu A_1^{(1)} + q_\mu A_2^{(1)}, \quad \hat{N}_2 \rightarrow Z_2, \\
\hat{p}_{1\mu} \hat{N}_2 &\rightarrow q_\mu \left[A_2^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_1^{(2)} \right],
\end{aligned}$$

where the symbol \doteq in the above equation reminds us that it is true only in the \mathcal{B}_μ^{PP} integration. $A_j^{(i)}$ and Z_2 , which are functions of $x_{1,2}$, $p_\perp'^2$, $p'_\perp \cdot q_\perp$, and q^2 , are listed in Appendix A. After the replacements, we arrive at

$$\begin{aligned}
f_+(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h_P' h_P''}{x_2 \hat{N}_1' \hat{N}_1''} \\
&\times \left[x_1 (M_0^{\prime 2} + M_0^{\prime\prime 2}) + x_2 q^2 - x_2 (m_1' - m_1'')^2 \right. \\
&\quad \left. - x_1 (m_1' - m_2)^2 - x_1 (m_1'' - m_2)^2 \right], \\
f_-(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h_P' h_P''}{x_2 \hat{N}_1' \hat{N}_1''} \\
&\times \left\{ -x_1 x_2 M^{\prime 2} - p_\perp'^2 - m_1' m_2 + (m_1'' - m_2) \right. \\
&\quad \left. \times (x_2 m_1' + x_1 m_2) + 2 \frac{q \cdot P}{q^2} \left(p_\perp'^2 + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
&+ 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \\
&- \frac{p'_\perp \cdot q_\perp}{q^2} \left[M^{\prime\prime 2} - x_2 (q^2 + q \cdot P) - (x_2 - x_1) M^{\prime 2} \right. \\
&\quad \left. + 2x_1 M_0^{\prime 2} - 2(m_1' - m_2)(m_1' + m_1'') \right] \Bigg\}. \quad (17)
\end{aligned}$$

Finally we get the form factors through the relations

$$F_1^{PP}(q^2) = f_+(q^2), \quad F_0^{PP}(q^2) = f_+(q^2) + \frac{q^2}{q \cdot P} f_-(q^2). \quad (18)$$

Similarly, one can derive the other form factors, whose expressions are collected in Appendix A.

Before closing this section, it is worth mentioning that the axial-vector mesons may not be classified as 3P_1 or 1P_1 state. In the quark limit with $m_Q \rightarrow \infty$, the QCD interaction is independent of the heavy quark spin and thus it will decouple with the light system. A consequence of this decoupling is that heavy mesons are classified into multiplets labeled by the total angular momentum of the light degrees of freedom. The s -wave pseudoscalar and vector states are in the same multiplets denoted as $s_l = 1/2$. For the p -wave states, two kinds of axial-vector mesons $P_1^{3/2}$ and $P_1^{1/2}$ are mixtures of 3P_1 or 1P_1 :

$$\begin{aligned}
|P_1^{3/2}\rangle &= \sqrt{\frac{2}{3}} |^1P_1\rangle + \sqrt{\frac{1}{3}} |^3P_1\rangle, \\
|P_1^{1/2}\rangle &= \sqrt{\frac{1}{3}} |^1P_1\rangle - \sqrt{\frac{2}{3}} |^3P_1\rangle.
\end{aligned} \quad (19)$$

Since the form factors involving $P_1^{3/2}$ and $P_1^{1/2}$ can be straightforwardly obtained by the linear combination for those given above, we shall calculate the form factors using the $^{2S+1}L_J$ basis in the following analysis.

3 Numerical results for form factors

3.1 Input parameters

In the covariant LFQM, the constituent quark masses are used as (in units of GeV):

$$m_u = m_d = 0.25, \quad m_s = 0.37, \quad m_c = 1.4, \quad m_b = 4.8, \quad (20)$$

which have been widely used in various B and B_c decays [42–49]. The masses of the B_c and B_{sJ} are taken from the PDG (in units of GeV) [11]:

$$m_{B_c} = 6.276, \quad m_{B_s} = 5.367, \quad m_{B_s^*} = 5.415, \\ m_{B_{s2}} = 5.840, \quad (21)$$

while for the B_{s0} and B_{s1} , we quote the results [51,52] (see also estimates in Refs. [53–55]):

$$m_{B_{s0}} = 5.782, \quad m_{B_{s1}(P_1^{1/2})} = 5.843, \quad m_{B_{s1}(P_1^{3/2})} = 5.833. \quad (22)$$

Since the masses of the $P_1^{1/2}$ and $P_1^{3/2}$ are close to the observed state $B_{s1}(5830)$ [11],

$$m_{B_{s1}(5830)} = 5.829 \text{ GeV}, \quad (23)$$

we use the same value for both the 3P_1 and the 1P_1 state.

The parameter β , characterizing the momentum distribution, is usually determined by fitting the meson decay constant. For instance, in this approach the pseudoscalar and vector meson's decay constants read

$$f_P = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P}{x_1 x_2 (M'^2 - M_0'^2)} 4(m'_1 x_2 + m_2 x'_1), \\ f_V = \frac{N_c}{4\pi^3 M'} \int dx_2 d^2 p'_\perp \frac{h'_V}{x_1 x_2 (M'^2 - M_0'^2)} \\ \times \left[x_1 M_0'^2 - m'_1(m'_1 - m_2) - p_\perp'^2 + \frac{m'_1 + m_2}{w'_V} p_\perp'^2 \right]. \quad (24)$$

For the B_c meson, the decay constant can in principle be determined by leptonic and radiative-leptonic decays [56–59], both of which show lack of experimental data yet. Two-loop contributions in the NRQCD framework have been calculated in Ref. [58] and the authors have found

$$f_{B_c} = 398 \text{ MeV}. \quad (25)$$

We will adopt this result, but it is necessary to note that the above value is smaller than the lattice QCD result by approximately 2σ : $f_{B_c} = (434 \pm 15) \text{ MeV}$. We use the recent lattice QCD result for the B_s decay constant with $N_f = 2 + 1 + 1$ [60],

$$f_{B_s} = (229 \pm 5) \text{ MeV}. \quad (26)$$

This is close to the previous lattice QCD result [61,62]: $f_{B_s} = (224 \pm 5) \text{ MeV}$. Using the decay constants, the shape parameters are fixed as

$$\beta_{B_c} = 0.886 \text{ GeV}, \quad \beta_{B_s} = 0.623 \text{ GeV}, \quad (27)$$

and we assume that the values of β for the other B_{sJ} mesons are approximately equal to that for the B_s , that is,

$$\beta_{B_s^*} = \beta_{B_{s0}} = \beta_{B_{s1}} = \beta_{B_{s1}'} = \beta_{B_{s2}} = 0.623 \text{ GeV}. \quad (28)$$

Table 2 $B_c \rightarrow B_s, B_s^*, B_{s0}, B_{s1}, B_{s1}',$ and B_{s2} form factors in the light-front quark model, which are fitted using Eq. (32), while for the form factors with an asterisk, the parametrization in Eq. (36) is adopted

F	$F(0)$	m_{fit}	δ	F	$F(0)$	m_{fit}	δ
$F_1^{B_c B_s}$	0.73	1.57	0.49	$F_0^{B_c B_s}$	0.73	2.07	0.82
$V^{B_c B_s^*}$	3.70	1.57	0.48	$A_0^{B_c B_s^*}$	0.55	1.49	0.61
$A_1^{B_c B_s^*}$	0.52	1.90	0.56	$A_2^{B_c B_s^*}$	0.07*	1.04*	0.37*
$F_1^{B_c B_{s0}}$	0.71	1.69	0.48	$F_0^{B_c B_{s0}}$	0.72*	1.98*	1.43*
$A^{B_c B_{s1}}$	0.19	1.71	0.45	$V_0^{B_c B_{s1}}$	0.10*	0.75*	0.95*
$V_1^{B_c B_{s1}}$	5.28*	2.28*	2.08*	$V_2^{B_c B_{s1}}$	0.07	1.73	0.32
$A^{B_c B_{s1}'}$	0.05	1.58	0.51	$V_0^{B_c B_{s1}'}$	0.63	1.76	0.60
$V_1^{B_c B_{s1}'}$	10.30	1.71	0.48	$V_2^{B_c B_{s1}'}$	-0.23	1.49	0.49
$V^{B_c B_{s2}}$	18.60	1.50	0.48	$A_0^{B_c B_{s2}}$	2.94	1.47	0.54
$A_1^{B_c B_{s2}}$	2.89	1.75	0.48	$A_2^{B_c B_{s2}}$	1.32*	3.24*	9.56*

We will also calculate the $B_c \rightarrow B_J$ form factors, for which we use the masses [51,52]

$$m_B = 5.279 \text{ GeV}, \quad m_{B^*} = 5.325 \text{ GeV}, \quad m_{B_0} = 5.749 \text{ GeV}, \\ m_{B_1} = m_{B_1'} = 5.731 \text{ GeV}, \quad m_{B_2} = 5.746 \text{ GeV}, \quad (29)$$

and the shape parameter β for the B_J meson:

$$\beta_{B_J} = 0.562 \text{ GeV}. \quad (30)$$

The above result is derived from the decay constant result [60],

$$f_B = (193 \pm 6) \text{ MeV}. \quad (31)$$

3.2 Form factors and momentum transfer distribution

With the inputs in the previous subsection, we can predict the $B_c \rightarrow B_s, B_s^*, B_{s0}, B_{s1}, B_{s1}',$ and B_{s2} form factors in the LFQM and we show our results in Table 2. In order to access the q^2 distribution, one may adopt the fit formula:

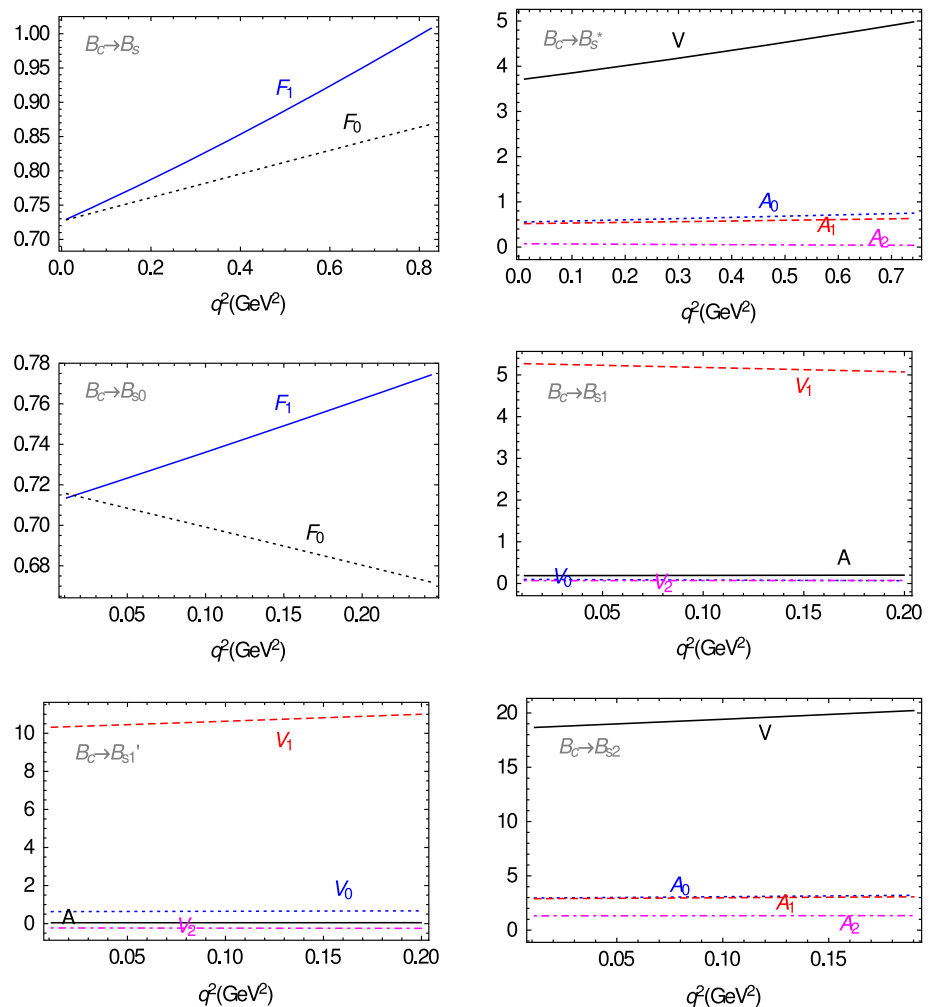
$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\text{fit}}^2} + \delta(\frac{q^2}{m_{\text{fit}}^2})^2}. \quad (32)$$

In the literature, the dipole form has been used to parametrize the q^2 distribution:

$$F(q^2) = \frac{F(0)}{1 - a\frac{q^2}{m_H^2} + b(\frac{q^2}{m_H^2})^2}, \quad (33)$$

with the $m_H = m_D$ for D decays and $m_H = m_B$ for B decays. This parametrization is inspired by analyticity. Taking the $F_1^{B \rightarrow \pi}$ as an example, we consider the timelike matrix element

Fig. 2 The $B_c \rightarrow B_s$ form factors and their q^2 -dependence parameterized by Eqs. (32) and (36)



$$\langle 0 | \bar{u} \gamma^\mu b | \pi(-p_\pi) \bar{B}(p_B) \rangle \sim \int \frac{d^4 q}{(2\pi)^4} \frac{i}{q^2 - m_X^2} \times \langle 0 | \bar{u} \gamma^\mu b | X \rangle \langle X | \pi(-p_\pi) \bar{B}(p_B) \rangle, \quad (34)$$

where the one-particle contribution has been singled out. The lowest resonance that can contribute is the vector B^* . This leads to the pole structure at large q^2 :

$$F_1^{B \rightarrow \pi}(q^2) \sim \frac{F_1(0)}{1 - q^2/m_{B^*}^2}. \quad (35)$$

Except the pole at m_{B^*} , there are residual dependences on q^2 , which can be effectively incorporated into the a, b of the dipole parametrization as shown in Eq. (33). However, for the $B_c \rightarrow B_{sJ}$ transition, one cannot simply apply Eq. (33), since the contributing states are the D_s resonances. Using the $m_H = m_{B_c}$ will not only disguise the genuine poles, but also lead to irrationally large results for parameters a and b . So in order to avoid this problem, we have adopted the parametrization in Eq. (32). From the results in Table 2, one can see that the m_{fit} for most form factors is between 1.5 and 2.0 GeV, close to the mass of a D_{sJ} resonance. This has validated our parametrization.

For the $A_2^{B_c B_s^*}$, $F_0^{B_c B_{s0}}$, $V_{0,1}^{B_c B_{s1}}$, and $A_2^{B_c B_{s2}}$, we found that the fitted values for the m_{fit}^2 are negative, and thus we use the following formula:

$$F(q^2) = \frac{F(0)}{1 + \frac{q^2}{m_{\text{fit}}^2} + \delta \left(\frac{q^2}{m_{\text{fit}}^2} \right)^2}. \quad (36)$$

The q^2 -dependent form factors of $B_c \rightarrow B_s$ are shown in Fig. 2. From this figure, we can see that except for the $B_c \rightarrow B_{s1}$ transition, most form factors are rather stable against the variation of q^2 . This is partly because of the limited phase space. This will also lead to a reliable prediction for the branching fractions given in the next section.

4 Phenomenological applications

4.1 Semileptonic B_c decays

The decay width for semileptonic decays of $B_c \rightarrow M \bar{l} \nu$, where $M = P, V, S, A, T$, can be derived by dividing the

Table 3 Branching fractions for $B_c \rightarrow B_{sJ} \bar{\ell} \nu$ using the $B_c \rightarrow B_{sJ}$ form factors given in Table 2. Here $\ell = e, \mu$

$\ell = e$	$\mathcal{B}_{\text{total}}$	$\mathcal{B}_L/\mathcal{B}_T$	$\ell = \mu$	$\mathcal{B}_{\text{total}}$	$\mathcal{B}_L/\mathcal{B}_T$
$B_c \rightarrow B_s \bar{\ell} \nu$	1.51×10^{-2}	–	$B_c \rightarrow B_s \bar{\ell} \nu$	1.43×10^{-2}	–
$B_c \rightarrow B_s^* \bar{\ell} \nu$	1.96×10^{-2}	1.13	$B_c \rightarrow B_s^* \bar{\ell} \nu$	1.83×10^{-2}	1.10
$B_c \rightarrow B_{s0} \bar{\ell} \nu$	6.58×10^{-4}	–	$B_c \rightarrow B_{s0} \bar{\ell} \nu$	5.23×10^{-4}	–
$B_c \rightarrow B_{s1} \bar{\ell} \nu$	8.31×10^{-5}	0.57	$B_c \rightarrow B_{s1} \bar{\ell} \nu$	6.33×10^{-5}	0.52
$B_c \rightarrow B'_{s1} \bar{\ell} \nu$	5.38×10^{-4}	2.38	$B_c \rightarrow B'_{s1} \bar{\ell} \nu$	3.98×10^{-4}	2.09
$B_c \rightarrow B_{s2} \bar{\ell} \nu$	2.98×10^{-5}	2.29	$B_c \rightarrow B_{s2} \bar{\ell} \nu$	1.97×10^{-5}	1.97

decay amplitude into hadronic part and leptonic part, both of which are Lorentz invariant so that can be readily evaluated. Then the differential decay widths for $B_c \rightarrow P \bar{\ell} \nu$ and $B_c \rightarrow V \bar{\ell} \nu$ turn out to be

$$\frac{d\Gamma(B_c \rightarrow P \bar{\ell} \nu)}{dq^2} = (1 - \hat{m}_l^2)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_P^2, q^2)} G_F^2 |V_{CKM}|^2}{384 m_{B_c}^3 \pi^3} \times \left\{ (\hat{m}_l^2 + 2) \lambda(m_{B_c}^2, m_P^2, q^2) F_1^2(q^2) + 3 \hat{m}_l^2 (m_{B_c}^2 - m_P^2)^2 F_0^2(q^2) \right\}, \quad (37)$$

$$\frac{d\Gamma_L(B_c \rightarrow V \bar{\ell} \nu)}{dq^2} = (1 - \hat{m}_l^2)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)} G_F^2 |V_{CKM}|^2}{384 m_{B_c}^3 \pi^3} \times \left\{ 3 \hat{m}_l^2 \lambda(m_{B_c}^2, m_V^2, q^2) A_0^2(q^2) + (\hat{m}_l^2 + 2) \times \left| \frac{1}{2m_V} \left[(m_{B_c}^2 - m_V^2 - q^2)(m_{B_c} + m_V) A_1(q^2) - \frac{\lambda(m_{B_c}^2, m_V^2, q^2)}{m_{B_c} + m_V} A_2(q^2) \right] \right|^2 \right\}, \quad (38)$$

$$\frac{d\Gamma^\pm(B_c \rightarrow V \bar{\ell} \nu)}{dq^2} = (1 - \hat{m}_l^2)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)} G_F^2 |V_{CKM}|^2}{384 m_{B_c}^3 \pi^3} \times \left\{ (m_l^2 + 2q^2) \lambda(m_{B_c}^2, m_V^2, q^2) \times \left| \frac{V(q^2)}{m_{B_c} + m_V} \mp \frac{(m_{B_c} + m_V) A_1(q^2)}{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}} \right|^2 \right\}, \quad (39)$$

where the superscript $+$ ($-$) denotes the right-handed (left-handed) polarizations of the vector mesons. We have $\lambda(m_{B_c}^2, m_i^2, q^2) = (m_{B_c}^2 + m_i^2 - q^2)^2 - 4m_{B_c}^2 m_i^2$ with $i = P, V$. $\hat{m}_l = m_l/\sqrt{q^2}$. The combined transverse and total differential decay widths are given by

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \quad (40)$$

The differential decay widths for $B_c \rightarrow S \bar{\ell} \nu$ and $B_c \rightarrow A \bar{\ell} \nu$ can be obtained by making the following replacements in the above expressions for $B_c \rightarrow P \bar{\ell} \nu$ and $B_c \rightarrow V \bar{\ell} \nu$:

$$m_P \rightarrow m_S, \quad F_i^{B_c P}(q^2) \rightarrow F_i^{B_c S}(q^2), \quad i = 0, 1, \quad (41)$$

and

$$m_{B_c} + m_V \rightarrow m_{B_c} - m_A, \quad V^{B_c V}(q^2) \rightarrow A^{B_c A}(q^2), \quad (42)$$

$$A_i^{B_c V}(q^2) \rightarrow V_i^{B_c A}(q^2), \quad i = 0, 1, 2, \quad (43)$$

respectively. The $d\Gamma_L/dq^2$ and $d\Gamma^\pm/dq^2$ for $B_c \rightarrow T \bar{\ell} \nu$ is given by Eq. (38) multiplied by $(\sqrt{\frac{2}{3}} \frac{|\vec{p}_T|}{m_T})^2$ and Eq. (39) multiplied by $(\frac{1}{\sqrt{2}} \frac{|\vec{p}_T|}{m_T})^2$, respectively. Here the \vec{p}_T denotes the momentum of the tensor meson in the B_c rest frame and m_T is mass of the tensor meson.

For the B_{sJ} final state, the inputs are form factors given in Table 2 and the masses of B_c and B_{sJ} s given in Eq. (21). The other input parameters are given as follows [11]:

$$\tau_{B_c} = (0.452 \times 10^{-12}) \text{ s}, \quad m_e = 0.511 \text{ MeV}, \quad m_\mu = 0.106 \text{ GeV}, \quad m_\tau = 1.78 \text{ GeV}, \quad G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}, \quad |V_{cs}| = 0.973, \quad (44)$$

and our predictions for the branching fractions are given in Table 3. It should be mentioned that in the above calculation we have considered $B_{s1}(B'_{s1})$ and $B_1(B'_1)$ to be in the $^3P_1(^1P_1)$ eigenstates.

For the B_J final state, we need to evaluate the form factors for $B_c \rightarrow B_J$ by following the same method, and our results are given in Table 4. The masses of B_c and B_J s are also given in Eqs. (21) and (29). The other inputs are the same as Eq. (44) but with $|V_{cs}| = 0.973$ replaced by $|V_{cd}| = 0.225$ [11]. With these inputs, our results for the branching fractions and ratios are given in Table 5.

From these tables, we can see that the branching fractions for $B_c \rightarrow B_s \bar{\ell} \nu$ and $B_c \rightarrow B_s^* \bar{\ell} \nu$ are at the percent level, while those for the $B_c \rightarrow B \bar{\ell} \nu$ and $B_c \rightarrow B^* \bar{\ell} \nu$ are suppressed by one order of magnitude. This is consistent with the results in the literature [12–20]. Branching fractions for channels with p -wave bottomed mesons in the final state

Table 4 $B_c \rightarrow B, B^*, B_0, B_1, B'_1$, and B_2 form factors in the covariant LFQM fitted through Eq. (32), except for the form factors with an asterisk, which are fitted using Eq. (36)

F	$F(0)$	m_{fit}	δ	F	$F(0)$	m_{fit}	δ
$F_1^{B_c B}$	0.64	1.50	0.52	$F_0^{B_c B}$	0.64	1.94	0.83
$V^{B_c B^*}$	3.44	1.50	0.51	$A_0^{B_c B^*}$	0.47	1.42	0.68
$A_1^{B_c B^*}$	0.44	1.84	0.63	$A_2^{B_c B^*}$	0.07*	1.03*	0.37*
$F_1^{B_c B_0}$	0.69	1.61	0.51	$F_0^{B_c B_0}$	0.69*	2.83*	4.84*
$A^{B_c B_1}$	0.21	1.64	0.49	$V_0^{B_c B_1}$	0.13*	2.48*	51.50*
$V_1^{B_c B_1}$	4.97*	3.14*	6.49*	$V_2^{B_c B_1}$	0.09	1.64	0.38
$A^{B_c B'_1}$	0.06	1.51	0.55	$V_0^{B_c B'_1}$	0.64	1.66	0.64
$V_1^{B_c B'_1}$	8.05	1.62	0.51	$V_2^{B_c B'_1}$	-0.24	1.42	0.53
$V^{B_c B_2}$	17.60	1.43	0.52	$A_0^{B_c B_2}$	2.64	1.40	0.59
$A_1^{B_c B_2}$	2.59	1.68	0.52	$A_2^{B_c B_2}$	1.31*	3.13*	9.72*

range from 10^{-4} to 10^{-6} . In decays with large phase space, the electron and muon masses can introduce about a few percents to branching ratios. While for those with limited phase space like $B_c \rightarrow B_{s2} \bar{\ell} \nu$, the effects due to the lepton mass difference can reach 30%. We hope these predictions can be examined in the future on the experimental side.

4.2 Nonleptonic B_c decays

Since the main purpose of this work is to investigate the production of B_{sJ} , we will focus on the decay modes which can be controlled under the factorization approach. Such decay modes are usually dominated by tree operators with effective Hamiltonian

$$\mathcal{H}_{\text{eff}}(c \rightarrow s \bar{d}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \times \left\{ C_1 [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\alpha] + C_2 [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta] \right\}, \quad (45)$$

where C_1 and C_2 are the Wilson coefficients, and α and β denote the color indices.

Table 5 Branching ratios for $B_c \rightarrow B_J \bar{\ell} \nu$ ($\ell = e, \mu$) with the $B_c \rightarrow B_J$ form factors given in Table 4

$\ell = e$	$\mathcal{B}_{\text{total}}$	$\mathcal{B}_L/\mathcal{B}_T$	$\ell = \mu$	$\mathcal{B}_{\text{total}}$	$\mathcal{B}_L/\mathcal{B}_T$
$B_c \rightarrow B \bar{\ell} \nu$	1.04×10^{-3}	—	$B_c \rightarrow B \bar{\ell} \nu$	1.00×10^{-3}	—
$B_c \rightarrow B^* \bar{\ell} \nu$	1.34×10^{-3}	1.06	$B_c \rightarrow B^* \bar{\ell} \nu$	1.27×10^{-3}	1.04
$B_c \rightarrow B_0 \bar{\ell} \nu$	4.60×10^{-5}	—	$B_c \rightarrow B_0 \bar{\ell} \nu$	3.77×10^{-5}	—
$B_c \rightarrow B_1 \bar{\ell} \nu$	1.52×10^{-5}	0.52	$B_c \rightarrow B_1 \bar{\ell} \nu$	1.28×10^{-5}	0.50
$B_c \rightarrow B'_1 \bar{\ell} \nu$	7.70×10^{-5}	2.51	$B_c \rightarrow B'_1 \bar{\ell} \nu$	6.28×10^{-5}	2.29
$B_c \rightarrow B_2 \bar{\ell} \nu$	5.15×10^{-6}	2.22	$B_c \rightarrow B_2 \bar{\ell} \nu$	3.90×10^{-6}	2.00

With the definitions of the decay constants,

$$\langle \pi^+(p) | \bar{u} \gamma_\mu \gamma_5 d | 0 \rangle = -i f_\pi p_\mu, \quad (46)$$

one can expect the factorization formula to have the following forms:

$$i \mathcal{M}(B_c^+ \rightarrow B_s \pi^+) = N m_{B_c}^2 (1 - r_{B_s}^2) F_0^{B_c B_s}(m_\pi^2), \quad (47)$$

$$i \mathcal{M}(B_c^+ \rightarrow B_s^* \pi^+) = (-i) N \sqrt{\lambda(m_{B_c}^2, m_{B_s^*}^2, m_\pi^2)} A_0^{B_c B_s^*}(m_\pi^2), \quad (48)$$

$$i \mathcal{M}(B_c^+ \rightarrow B_{s0} \pi^+) = (-i) N m_{B_c}^2 (1 - r_{B_{s0}}^2) F_0^{B_c B_{s0}}(m_\pi^2), \quad (49)$$

$$i \mathcal{M}(B_c^+ \rightarrow B_{s1} \pi^+) = (-i) N \sqrt{\lambda(m_{B_c}^2, m_{B_{s1}}^2, m_\pi^2)} V_0^{B_c B_{s1}}(m_\pi^2), \quad (50)$$

$$i \mathcal{M}(B_c^+ \rightarrow B'_{s1} \pi^+) = (-i) N \sqrt{\lambda(m_{B_c}^2, m_{B'_{s1}}^2, m_\pi^2)} V_0^{B_c B'_{s1}}(m_\pi^2), \quad (51)$$

$$i \mathcal{M}(B_c^+ \rightarrow B_{s2} \pi^+) = (-i) \frac{1}{\sqrt{6}} N \frac{\lambda(m_{B_c}^2, m_{B_{s2}}^2, m_\pi^2)}{m_{B_c}^2 r_{B_{s2}}} A_0^{B_c B_{s2}}(m_\pi^2), \quad (52)$$

where $N = G_F / \sqrt{2} V_{cs}^* V_{ud} a_1 f_\pi$, with $a_1 = C_2 + C_1 / N_c$ ($N_c = 3$).

The particle decay width for $B_c \rightarrow B_{sJ} \pi$ is given as

$$\Gamma = \frac{|\vec{p}_1|}{8\pi m_{B_c}^2} |\mathcal{M}|^2 \quad (53)$$

with $|\vec{p}_1|$ being the magnitude of three-momentum of B_{sJ} or π meson in the final state in the B_c rest frame.

We use the transition form factors given in Table 2 and the masses of B_c and B_{sJ} s given in Eqs. (21), (22), and (23) and the other inputs which are given as follows [11, 44]:

$$\tau_{B_c} = (0.452 \times 10^{-12}) \text{ s}, \quad m_\pi = 0.140 \text{ GeV}, \quad |V_{cs}| = 0.973, \quad |V_{ud}| = 0.974, \quad (54)$$

$$f_\pi = 130.4 \text{ MeV}, \quad a_1 = 1.07, \quad (55)$$

where f_π can be extracted from $\pi^- \rightarrow \ell^- \bar{\nu}$ data and a_1 is evaluated at the typical factorization scale $\mu \sim m_c$ [63]. Then our theoretical results for the $B_c \rightarrow B_{sJ} \pi$ branching ratios turn out to be as follows:

$$\begin{aligned}
\mathcal{B}(B_c^+ \rightarrow B_s \pi^+) &= 4.1 \%, \\
\mathcal{B}(B_c^+ \rightarrow B_s^* \pi^+) &= 2.0 \%, \\
\mathcal{B}(B_c^+ \rightarrow B_{s0} \pi^+) &= 0.68 \%, \\
\mathcal{B}(B_c^+ \rightarrow B_{s1} \pi^+) &= 0.0082 \%, \\
\mathcal{B}(B_c^+ \rightarrow B_{s1}' \pi^+) &= 0.36 \%, \\
\mathcal{B}(B_c^+ \rightarrow B_{s2} \pi^+) &= 0.023 \%.
\end{aligned} \quad (56)$$

Using the 1 fb^{-1} data of proton–proton collisions collected at the center-of-mass energy of 7 TeV and 2 fb^{-1} data accumulated at 8 TeV, the LHCb Collaboration has observed the decay $B_c \rightarrow B_s \pi^+$ [64]:

$$\begin{aligned}
\frac{\sigma(B_c^+)}{\sigma(B_s^0)} \times \mathcal{B}(B_c^+ \rightarrow B_s^0 \pi^+) \\
= (2.37 \pm 0.31 \pm 0.11_{-0.13}^{+0.17}) \times 10^{-3}.
\end{aligned} \quad (57)$$

The first uncertainty is statistical, the second is systematic, and the third arises from the uncertainty on the B_c^+ lifetime. The ratio of cross sections $\sigma(B_c^+)/\sigma(B_s^0)$ depends significantly on the kinematics, and a rough estimate has led to the branching ratio for $B_c^+ \rightarrow B_s^0 \pi^+$ of about 10 % [64]. The estimated branching fraction is somewhat larger than but still at the same magnitude as our result. Moreover, our results have indicated that the LHCb Collaboration might be able to discover other channels with similar branching fractions like the $B_c \rightarrow B_s^* \pi$.

5 Conclusions

To understand the structure of the heavy–light mesons, especially the newly observed states, and to establish an overview of the spectroscopy, a lot of effort is required on both the experiment and the theory sides. One particular remark is the classification of these states. In the heavy quark limit, the charm quark will decouple with the light degree of freedom and acts as a static color source. Strong interactions will be independent of the heavy flavor and spin. In this case, heavy mesons, the eigenstates of the QCD Lagrangian in the heavy quark limit, can be labeled according to the total angular momentum s_l of the light degree of freedom. The heavy mesons with the same angular momentum s_l but different orientations of the heavy quark spin degenerate. One consequence is that heavy mesons can be classified by the multiplets characterized by s_l instead of the usual scheme using the $^{2S+1}L_J$.

In this work, we have suggested to study the B_s and its excitations B_{sJ} in the B_c decays. We have calculated the $B_c \rightarrow B_{sJ}$ and $B_c \rightarrow B_J$ form factors within the covariant light-front quark model, where the B_{sJ} and B_J denotes an s-wave or p-wave $\bar{b}s$ and $\bar{b}d$ meson, respectively. The form factors at $q^2 = 0$ are directly calculated while the q^2 -distribution

is obtained by the extrapolation. The derived form factors are then used to study semileptonic $B_c \rightarrow (B_{sJ}, B_J) \bar{\ell} \nu$ decays, and nonleptonic $B_c \rightarrow B_{sJ} \pi$ decays. Branching fractions and polarizations are predicted, through which we find that the predicted branching fractions are sizable, especially at the LHC experiment and future high-energy e^+e^- colliders with a high luminosity at the Z-pole. The future experimental measurements are helpful to study the nonperturbative QCD dynamics in the presence of a heavy spectator and also they are of great value for the study of spectroscopy.

Acknowledgments This work is supported in part by National Natural Science Foundation of China under Grant No. 11575110, Natural Science Foundation of Shanghai under Grant No. 15DZ2272100 and No. 15ZR1423100, by the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No. Y5KF111CJ1), and by Scientific Research Foundation for Returned Overseas Chinese Scholars, State Education Ministry.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

Appendix A: Explicit expressions for form factors

In the LFQM, it is more convenient to adopt a new set of parametrization of form factors, with the relations

$$\begin{aligned}
V^{B_c V}(q^2) &= -(m_{B_c} + m_V)g(q^2), \quad A_1^{B_c V}(q^2) \\
&= -\frac{f(q^2)}{m_{B_c} + m_V}, \quad A_2^{B_c V}(q^2) = (m_{B_c} + m_V)a_+(q^2), \\
A_0^{B_c V}(q^2) &= \frac{m_{B_c} + m_V}{2m_V} A_1^{B_c V}(q^2) - \frac{m_{B_c} - m_V}{2m_V} A_2^{B_c V}(q^2) \\
&\quad - \frac{q^2}{2m_V} a_-(q^2),
\end{aligned} \quad (A1)$$

$$\begin{aligned}
F_1^{B_c S}(q^2) &= -u_+(q^2), \quad F_0^{B_c S}(q^2) = -u_+(q^2) \\
&\quad - \frac{q^2}{q \cdot P} u_-(q^2), \\
A^{B_c A}(q^2) &= -(m_{B_c} - m_A)q(q^2), \quad V_1^{B_c A}(q^2) \\
&= -\frac{\ell(q^2)}{m_{B_c} - m_A}, \quad V_2^{B_c A}(q^2) = (m_{B_c} - m_A)c_+(q^2), \\
V_0^{B_c A}(q^2) &= \frac{m_{B_c} - m_A}{2m_A} V_1^{B_c A}(q^2) - \frac{m_{B_c} + m_A}{2m_A} V_2^{B_c A}(q^2) \\
&\quad - \frac{q^2}{2m_A} c_-(q^2),
\end{aligned} \quad (A2)$$

$$\begin{aligned}
V^{B_c T}(q^2) &= -m_{B_c}(m_{B_c} + m_T)h(q^2), \quad A_1^{B_c T}(q^2) \\
&= -\frac{m_{B_c}}{m_{B_c} + m_T} k(q^2), \quad A_2^{B_c T}(q^2) = m_{B_c}(m_{B_c} + m_T)b_+(q^2),
\end{aligned}$$

$$A_0^{B_c T}(q^2) = \frac{m_{B_c} + m_T}{2m_T} A_1^{B_c T}(q^2) - \frac{m_{B_c} - m_T}{2m_T} A_2^{B_c T}(q^2) - \frac{m_{B_c} q^2}{2m_T} b_-(q^2). \quad (A3)$$

The analytic expressions for $P \rightarrow P$ transition form factors in the covariant LFQM have been given in Eq. (17), while for the $P \rightarrow V$ transition, they are given as follows [31, 37]:

$$g(q^2) = -\frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ x_2 m'_1 + x_1 m_2 + (m'_1 - m''_1) \frac{p'_\perp \cdot q_\perp}{q^2} + \frac{2}{w''_V} \left[p_\perp'^2 + \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right] \right\}, \quad (A4)$$

$$f(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \times \left\{ 2x_1(m_2 - m'_1)(M_0'^2 + M_0''^2) - 4x_1 m''_1 M_0'^2 + 2x_2 m'_1 q \cdot P + 2m_2 q^2 - 2x_1 m_2 (M'^2 + M''^2) + 2(m'_1 - m_2)(m'_1 + m''_1)^2 + 8(m'_1 - m_2) \left[p_\perp'^2 + \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right] + 2(m'_1 + m''_1)(q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2} - 4 \frac{q^2 p_\perp'^2 + (p'_\perp \cdot q_\perp)^2}{q^2 w''_V} \left[2x_1 (M'^2 + M_0'^2) - q^2 - q \cdot P - 2(q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2} - 2(m'_1 - m''_1)(m'_1 - m_2) \right] \right\}, \quad (A5)$$

$$a_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \times \left\{ (x_1 - x_2)(x_2 m'_1 + x_1 m_2) - [2x_1 m_2 + m''_1 + (x_2 - x_1) m'_1] \frac{p'_\perp \cdot q_\perp}{q^2} - 2 \frac{x_2 q^2 + p'_\perp \cdot q_\perp}{x_2 q^2 w''_V} [p'_\perp \cdot p''_\perp + (x_1 m_2 + x_2 m'_1)(x_1 m_2 - x_2 m''_1)] \right\}, \quad (A6)$$

$$a_-(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_V}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ 2(2x_1 - 3) \times (x_2 m'_1 + x_1 m_2) - 8(m'_1 - m_2) \left[\frac{p_\perp'^2}{q^2} + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^4} \right] - [(14 - 12x_1)m'_1 - 2m''_1 - (8 - 12x_1)m_2] \frac{p'_\perp \cdot q_\perp}{q^2} + \frac{4}{w''_V} \left[[M'^2 + M''^2 - q^2 + 2(m'_1 - m''_1)(m'_1 - m_2)] \right. \right.$$

$$\times (A_3^{(2)} + A_4^{(2)} - A_2^{(1)}) + Z_2(3A_2^{(1)} - 2A_4^{(2)} - 1) + \frac{1}{2} [x_1(q^2 + q \cdot P) - 2M'^2 - 2p'_\perp \cdot q_\perp - 2m'_1(m''_1 + m_2) - 2m_2(m'_1 - m_2)] (A_1^{(1)} + A_2^{(1)} - 1) + q \cdot P \left[\frac{p_\perp'^2}{q^2} + \frac{(p'_\perp \cdot q_\perp)^2}{q^4} \right] (4A_2^{(1)} - 3) \left. \right\}. \quad (A7)$$

The explicit expressions for $P \rightarrow S$ and $P \rightarrow A$ transitions can be readily obtained by making the following replacements [37]:

$$u_\pm(q^2) = -f_\pm(q^2)|_{m'_1 \rightarrow -m''_1, h'_P \rightarrow h''_S}, \\ [\ell^3 A, {}^1 A(q^2), q^3 A, {}^1 A(q^2), c^3 A, {}^1 A(q^2)] \\ = [f(q^2), g(q^2), a_\pm(q^2)]|_{m'_1 \rightarrow -m''_1, h'_V \rightarrow h''_{3A, {}^1 A}, w''_V \rightarrow w''_{3A, {}^1 A}}, \quad (A8)$$

where only the $1/W''$ terms in $P \rightarrow {}^1 A$ form factors are kept. It should be cautious that the replacement of $m'_1 \rightarrow -m''_1$ should not be applied to m'_1 in w'' and h'' . The $P \rightarrow T$ transition form factors are calculated [37]

$$h(q^2) = -g(q^2)|_{h'_V \rightarrow h''_T} + \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_P h''_T}{x_2 \hat{N}'_1 \hat{N}''_1} \times \left\{ (m'_1 - m''_1)(A_3^{(2)} + A_4^{(2)}) + (m''_1 + m'_1 - 2m_2)(A_2^{(2)} + A_3^{(2)}) - m'_1(A_1^{(1)} + A_2^{(1)}) + \frac{2}{w''_V} (2A_1^{(3)} + 2A_2^{(3)} - A_1^{(2)}) \right\}, \quad (A9)$$

$$k(q^2) = -f(q^2)|_{h'_V \rightarrow h''_T} + \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_T}{x_2 \hat{N}'_1 \hat{N}''_1} \times \left\{ 2(A_1^{(1)} + A_2^{(1)}) \times [m_2(q^2 - \hat{N}'_1 - \hat{N}''_1 - m_1'^2 - m_1''^2) - m'_1(M''^2 - \hat{N}'_1 - m_1''^2 - m_2^2) - m''_1(M'^2 - \hat{N}'_1 - m_1'^2 - m_2^2) - 2m'_1 m''_1 m_2] + 2(m'_1 + m''_1) \times \left(A_2^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_1^{(2)} \right) + 16(m_2 - m'_1)(A_1^{(3)} + A_2^{(3)}) + 4(2m'_1 - m''_1 - m_2) A_1^{(2)} + \frac{4}{w''_V} \left([M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2)] \times (2A_1^{(3)} + 2A_2^{(3)} - A_1^{(2)}) - 4 \left[A_2^{(3)} Z_2 + \frac{q \cdot P}{3q^2} (A_1^{(2)})^2 \right] + 2A_1^{(2)} Z_2 \right) \right\}, \quad (A10)$$

$$b_+(q^2) = -a_+(q^2)|_{h'_V \rightarrow h''_T} + \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_T}{x_2 \hat{N}'_1 \hat{N}''_1}$$

$$\begin{aligned}
& \times \left\{ 8(m_2 - m'_1)(A_3^{(3)} + 2A_4^{(3)} + A_5^{(3)}) \right. \\
& - 2m'_1(A_1^{(1)} + A_2^{(1)})(A_2^{(2)} + A_3^{(2)}) \\
& + 2(m'_1 + m''_1)(A_2^{(2)} + 2A_3^{(2)} + A_4^{(2)}) \\
& + \frac{2}{w_V''} [2(M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2))] \\
& \times (A_3^{(3)} + 2A_4^{(3)} + A_5^{(3)} - A_2^{(2)} - A_3^{(2)}) \\
& + [q^2 - \hat{N}'_1 - \hat{N}''_1 - (m'_1 + m''_1)^2] \\
& \left. \times (A_2^{(2)} + 2A_3^{(2)} + A_4^{(2)} - A_1^{(1)} - A_2^{(1)}) \right\}, \quad (A11)
\end{aligned}$$

$$\begin{aligned}
b_-(q^2) = & -a_-(q^2)|_{h_V'' \rightarrow h_T''} + \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_p h''_T}{x_2 \hat{N}'_1 \hat{N}''_1} \\
& \times \left\{ 8(m_2 - m'_1)(A_4^{(3)} + 2A_5^{(3)} + A_6^{(3)}) \right. \\
& - 6m'_1(A_1^{(1)} + A_2^{(1)}) + 4(2m'_1 - m''_1 - m_2)(A_3^{(2)} + A_4^{(2)}) \\
& + 2(3m'_1 + m''_1 - 2m_2)(A_2^{(2)} + 2A_3^{(2)} + A_4^{(2)}) \\
& + \frac{2}{w_V''} [2(M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2))] \\
& \times (A_4^{(3)} + 2A_5^{(3)} + A_6^{(3)} - A_3^{(2)} - A_4^{(2)}) + 2Z_2(3A_4^{(2)} \\
& - 2A_6^{(3)} - A_2^{(1)}) \\
& + 2\frac{q \cdot P}{q^2} \left(6A_2^{(1)} A_1^{(2)} - 6A_2^{(1)} A_2^{(3)} + \frac{2}{q^2} (A_1^{(2)})^2 - A_1^{(2)} \right) \\
& + [q^2 - 2M'^2 + \hat{N}'_1 - \hat{N}''_1 - (m'_1 + m''_1)^2 + 2(m'_1 - m_2)^2] \\
& \left. \times (A_2^{(2)} + 2A_3^{(2)} + A_4^{(2)} - A_1^{(1)} - A_2^{(1)}) \right\}. \quad (A12)
\end{aligned}$$

The $A_j^{(i)}$ in the above equations are given as follows:

$$\begin{aligned}
A_1^{(1)} &= \frac{x_1}{2}, \quad A_2^{(1)} = A_1^{(1)} - \frac{p'_\perp \cdot q_\perp}{q^2}, \\
A_1^{(2)} &= -p_\perp'^2 - \frac{(p'_\perp \cdot q_\perp)^2}{q^2}, \\
A_2^{(2)} &= (A_1^{(1)})^2, \quad A_3^{(2)} = A_1^{(1)} A_2^{(1)}, \\
A_4^{(2)} &= (A_2^{(1)})^2 - \frac{1}{q^2} A_1^{(2)}, \quad A_1^{(3)} = A_1^{(1)} A_1^{(2)}, \\
A_2^{(3)} &= A_2^{(1)} A_1^{(2)}, \quad A_3^{(3)} = A_1^{(1)} A_2^{(2)}, \quad A_4^{(3)} = A_2^{(1)} A_2^{(2)}, \\
A_5^{(3)} &= A_1^{(1)} A_4^{(2)}, \quad A_6^{(3)} = A_2^{(1)} A_4^{(2)} - \frac{2}{q^2} A_2^{(1)} A_1^{(2)}, \\
Z_2 &= \hat{N}'_1 + m_1'^2 - m_2^2 + (1 - 2x_1)M'^2 \\
&+ (q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2}. \quad (A13)
\end{aligned}$$

The explicit forms of h'_M and w'_M are given by [37]

$$\begin{aligned}
h'_P &= h'_V = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}'_0} \varphi', \\
h'_S &= \sqrt{\frac{2}{3}} h'_{3A} = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}'_0} \frac{\tilde{M}'_0'^2}{2\sqrt{3} M'_0} \varphi'_p, \\
h'_{1A} &= h'_T = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}'_0} \varphi'_p \\
w'_V &= M'_0 + m'_1 + m_2, \quad w'_{3A} = \frac{\tilde{M}'_0'^2}{m'_1 - m_2}, \quad w'_{1A} = 2, \quad (A14)
\end{aligned}$$

where φ' and φ'_p are the light-front momentum distribution amplitudes for s -wave and p -wave mesons, respectively [37]:

$$\begin{aligned}
\varphi' &= \varphi'(x_2, p'_\perp) = 4 \left(\frac{\pi}{\beta'^2} \right)^{3/4} \sqrt{\frac{dp'_z}{dx_2}} \exp \left(-\frac{p_z'^2 + p_\perp'^2}{2\beta'^2} \right), \\
\varphi'_p &= \varphi'_p(x_2, p'_\perp) = \sqrt{\frac{2}{\beta'^2}} \varphi', \quad \frac{dp'_z}{dx_2} = \frac{e'_1 e_2}{x_1 x_2 M'_0}. \quad (A15)
\end{aligned}$$

References

1. S.K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003). doi:[10.1103/PhysRevLett.91.262001](https://doi.org/10.1103/PhysRevLett.91.262001). [arXiv:hep-ex/0309032](https://arxiv.org/abs/hep-ex/0309032)
2. D. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. **93**, 072001 (2004). doi:[10.1103/PhysRevLett.93.072001](https://doi.org/10.1103/PhysRevLett.93.072001). [arXiv:hep-ex/0312021](https://arxiv.org/abs/hep-ex/0312021)
3. B. Aubert et al. [BaBar Collaboration], Phys. Rev. D **71**, 071103 (2005). doi:[10.1103/PhysRevD.71.071103](https://doi.org/10.1103/PhysRevD.71.071103). [arXiv:hep-ex/0406022](https://arxiv.org/abs/hep-ex/0406022)
4. V.M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004). doi:[10.1103/PhysRevLett.93.162002](https://doi.org/10.1103/PhysRevLett.93.162002). [arXiv:hep-ex/0405004](https://arxiv.org/abs/hep-ex/0405004)
5. N. Brambilla et al., Eur. Phys. J. C **71**, 1534 (2011). doi:[10.1140/epjc/s10052-010-1534-9](https://doi.org/10.1140/epjc/s10052-010-1534-9). [arXiv:1010.5827](https://arxiv.org/abs/1010.5827) [hep-ph]
6. A. Esposito, A.L. Guerrieri, F. Piccinini, A. Pilloni, A.D. Polosa, Int. J. Mod. Phys. A **30**, 1530002 (2015). doi:[10.1142/S0217751X15300021](https://doi.org/10.1142/S0217751X15300021). [arXiv:1411.5997](https://arxiv.org/abs/1411.5997) [hep-ph]
7. H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Phys. Rep. **639**, 1 (2016). doi:[10.1016/j.physrep.2016.05.004](https://doi.org/10.1016/j.physrep.2016.05.004). [arXiv:1601.02092](https://arxiv.org/abs/1601.02092) [hep-ph]
8. A. Ali, [arXiv:1605.05954](https://arxiv.org/abs/1605.05954) [hep-ph]
9. B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. **90**, 242001 (2003). doi:[10.1103/PhysRevLett.90.242001](https://doi.org/10.1103/PhysRevLett.90.242001). [arXiv:hep-ex/0304021](https://arxiv.org/abs/hep-ex/0304021)
10. D. Besson et al. [CLEO Collaboration], Phys. Rev. D **68**, 032002 (2003). Erratum: [Phys. Rev. D **75**, 119908 (2007)] doi:[10.1103/PhysRevD.68.032002](https://doi.org/10.1103/PhysRevD.68.032002), doi:[10.1103/PhysRevD.75.119908](https://doi.org/10.1103/PhysRevD.75.119908). [arXiv:hep-ex/0305100](https://arxiv.org/abs/hep-ex/0305100)
11. K.A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014). doi:[10.1088/1674-1137/38/9/090001](https://doi.org/10.1088/1674-1137/38/9/090001)
12. C.H. Chang, Y.Q. Chen, Phys. Rev. D **49**, 3399 (1994). doi:[10.1103/PhysRevD.49.3399](https://doi.org/10.1103/PhysRevD.49.3399)
13. G.G. Lu, Y.D. Yang, H.B. Li, Phys. Lett. B **341**, 391 (1995). doi:[10.1016/0370-2693\(94\)01333-8](https://doi.org/10.1016/0370-2693(94)01333-8), doi:[10.1016/0370-2693\(95\)80020-X](https://doi.org/10.1016/0370-2693(95)80020-X)
14. J.F. Liu, K.T. Chao, Phys. Rev. D **56**, 4133 (1997). doi:[10.1103/PhysRevD.56.4133](https://doi.org/10.1103/PhysRevD.56.4133)

15. P. Colangelo, F. De Fazio, *Phys. Rev. D* **61**, 034012 (2000). doi:[10.1103/PhysRevD.61.034012](#). arXiv:[hep-ph/9909423](#)
16. M.A. Ivanov, J.G. Korner, P. Santorelli, *Phys. Rev. D* **63**, 074010 (2001). doi:[10.1103/PhysRevD.63.074010](#). arXiv:[hep-ph/0007169](#)
17. V.V. Kiselev, A.E. Kovalsky, A.K. Likhoded, *Nucl. Phys. B* **585**, 353 (2000). doi:[10.1016/S0550-3213\(00\)00386-2](#). arXiv:[hep-ph/0002127](#)
18. A. Abd El-Hady, J.H. Munoz, J.P. Vary, *Phys. Rev. D* **62**, 014019 (2000). doi:[10.1103/PhysRevD.62.014019](#). arXiv:[hep-ph/9909406](#)
19. D. Choudhury, A. Kundu, B. Mukhopadhyaya, *Mod. Phys. Lett. A* **16**, 1439 (2001). doi:[10.1142/S0217732301004650](#)
20. D. Ebert, R.N. Faustov, V.O. Galkin, *Eur. Phys. J. C* **32**, 29 (2003). doi:[10.1140/epjc/s2003-01347-5](#). arXiv:[hep-ph/0308149](#)
21. J.F. Sun, Y.L. Yang, W.J. Du, H.L. Ma, *Phys. Rev. D* **77**, 114004 (2008). doi:[10.1103/PhysRevD.77.114004](#). arXiv:[0806.1254](#) [hep-ph]
22. D. Ebert, R.N. Faustov, V.O. Galkin, *Phys. Rev. D* **82**, 034019 (2010). doi:[10.1103/PhysRevD.82.034019](#). arXiv:[1007.1369](#) [hep-ph]
23. H.F. Fu, Y. Jiang, C.S. Kim, G.L. Wang, *JHEP* **1106**, 015 (2011). doi:[10.1007/JHEP06\(2011\)015](#). arXiv:[1102.5399](#) [hep-ph]
24. S. Naimuddin, S. Kar, M. Priyadarsini, N. Barik, P.C. Dash, *Phys. Rev. D* **86**, 094028 (2012). doi:[10.1103/PhysRevD.86.094028](#)
25. Z.J. Xiao, X. Liu, *Chin. Sci. Bull.* **59**, 3748 (2014). doi:[10.1007/s11434-014-0418-z](#). arXiv:[1401.0151](#) [hep-ph]
26. J. Sun, Y. Yang, G. Lu, *Sci. China Phys. Mech. Astron.* **57**(10), 1891 (2014). doi:[10.1007/s11433-014-5535-9](#). arXiv:[1406.4927](#) [hep-ph]
27. J. Sun, Y. Yang, Q. Chang, G. Lu, *Phys. Rev. D* **89**(11), 114019 (2014). doi:[10.1103/PhysRevD.89.114019](#). arXiv:[1406.4925](#) [hep-ph]
28. J. Sun, N. Wang, Q. Chang, Y. Yang, *Adv. High Energy Phys.* **2015**, 104378 (2015). doi:[10.1155/2015/104378](#). arXiv:[1504.01286](#) [hep-ph]
29. X.G. He, W. Wang, R.L. Zhu, arXiv:[1606.00097](#) [hep-ph]
30. A. Choudhury, A. Kundu, B. Mukhopadhyaya, arXiv:[1606.08402](#) [hep-ph]
31. W. Jaus, *Phys. Rev. D* **60**, 054026 (1999). doi:[10.1103/PhysRevD.60.054026](#)
32. S.J. Brodsky, H.C. Pauli, S.S. Pinsky, *Phys. Rep.* **301**, 299 (1998). doi:[10.1016/S0370-1573\(97\)00089-6](#). arXiv:[hep-ph/9705477](#)
33. W. Jaus, *Phys. Rev. D* **41**, 3394 (1990). doi:[10.1103/PhysRevD.41.3394](#)
34. W. Jaus, *Phys. Rev. D* **44**, 2851 (1991). doi:[10.1103/PhysRevD.44.2851](#)
35. H.Y. Cheng, C.Y. Cheung, C.W. Hwang, *Phys. Rev. D* **55**, 1559 (1997). doi:[10.1103/PhysRevD.55.1559](#). arXiv:[hep-ph/9607332](#)
36. H.M. Choi, C.R. Ji, L.S. Kisslinger, *Phys. Rev. D* **65**, 074032 (2002). doi:[10.1103/PhysRevD.65.074032](#). arXiv:[hep-ph/0110222](#)
37. H.Y. Cheng, C.K. Chua, C.W. Hwang, *Phys. Rev. D* **69**, 074025 (2004). doi:[10.1103/PhysRevD.69.074025](#). arXiv:[hep-ph/0310359](#)
38. H.Y. Cheng, C.K. Chua, *Phys. Rev. D* **69**, 094007 (2004). Erratum: [*Phys. Rev. D* **81**, 059901 (2010)]. doi:[10.1103/PhysRevD.69.094007](#), doi:[10.1103/PhysRevD.81.059901](#). arXiv:[hep-ph/0401141](#)
39. H.W. Ke, X.Q. Li, Z.T. Wei, *Phys. Rev. D* **80**, 074030 (2009). doi:[10.1103/PhysRevD.80.074030](#). arXiv:[0907.5465](#) [hep-ph]
40. H.W. Ke, X.Q. Li, Z.T. Wei, *Eur. Phys. J. C* **69**, 133 (2010). doi:[10.1140/epjc/s10052-010-1383-6](#). arXiv:[0912.4094](#) [hep-ph]
41. H.Y. Cheng, C.K. Chua, *Phys. Rev. D* **81**, 114006 (2010). Erratum: [*Phys. Rev. D* **82**, 059904 (2010)]. doi:[10.1103/PhysRevD.81.114006](#), doi:[10.1103/PhysRevD.82.059904](#). arXiv:[0909.4627](#) [hep-ph]
42. C.D. Lu, W. Wang, Z.T. Wei, *Phys. Rev. D* **76**, 014013 (2007). doi:[10.1103/PhysRevD.76.014013](#). arXiv:[hep-ph/0701265](#)
43. W. Wang, Y.L. Shen, C.D. Lu, *Eur. Phys. J. C* **51**, 841 (2007). doi:[10.1140/epjc/s10052-007-0334-3](#). arXiv:[0704.2493](#) [hep-ph]
44. W. Wang, Y.L. Shen, C.D. Lu, *Phys. Rev. D* **79**, 054012 (2009). doi:[10.1103/PhysRevD.79.054012](#). arXiv:[0811.3748](#) [hep-ph]
45. W. Wang, Y.L. Shen, *Phys. Rev. D* **78**, 054002 (2008). doi:[10.1103/PhysRevD.78.054002](#)
46. X.X. Wang, W. Wang, C.D. Lu, *Phys. Rev. D* **79**, 114018 (2009). doi:[10.1103/PhysRevD.79.114018](#). arXiv:[0901.1934](#) [hep-ph]
47. C.H. Chen, Y.L. Shen, W. Wang, *Phys. Lett. B* **686**, 118 (2010). doi:[10.1016/j.physletb.2010.02.056](#). arXiv:[0911.2875](#) [hep-ph]
48. G. Li, F.I. Shao, W. Wang, *Phys. Rev. D* **82**, 094031 (2010). doi:[10.1103/PhysRevD.82.094031](#). arXiv:[1008.3696](#) [hep-ph]
49. R.C. Verma, *J. Phys. G* **39**, 025005 (2012). doi:[10.1088/0954-3899/39/2/025005](#). arXiv:[1103.2973](#) [hep-ph]
50. H.W. Ke, X.Q. Li, *Eur. Phys. J. C* **71**, 1776 (2011). doi:[10.1140/epjc/s10052-011-1776-1](#). arXiv:[1104.3996](#) [hep-ph]
51. J.B. Liu, M.Z. Yang, *Chin. Phys. C* **40**(7), 073101 (2016). doi:[10.1088/1674-1137/40/7/073101](#). arXiv:[1507.08372](#) [hep-ph]
52. J.B. Liu, C.D. Lu, arXiv:[1605.05550](#) [hep-ph]
53. M. Di Pierro, E. Eichten, *Phys. Rev. D* **64**, 114004 (2001). doi:[10.1103/PhysRevD.64.114004](#). arXiv:[hep-ph/0104208](#)
54. D. Ebert, R.N. Faustov, V.O. Galkin, *Eur. Phys. J. C* **66**, 197 (2010). doi:[10.1140/epjc/s10052-010-1233-6](#). arXiv:[0910.5612](#) [hep-ph]
55. Z.H. Wang, G.L. Wang, H.F. Fu, Y. Jiang, *Phys. Lett. B* **706**, 389 (2012). doi:[10.1016/j.physletb.2011.11.051](#). arXiv:[1202.1224](#) [hep-ph]
56. C.H. Chang, J.P. Cheng, C.D. Lu, *Phys. Lett. B* **425**, 166 (1998). doi:[10.1016/S0370-2693\(98\)00177-4](#). arXiv:[hep-ph/9712325](#)
57. C.H. Chang, C.D. Lu, G.L. Wang, H.S. Zong, *Phys. Rev. D* **60**, 114013 (1999). doi:[10.1103/PhysRevD.60.114013](#). arXiv:[hep-ph/9904471](#)
58. L.B. Chen, C.F. Qiao, *Phys. Lett. B* **748**, 443 (2015). doi:[10.1016/j.physletb.2015.07.043](#). arXiv:[1503.05122](#) [hep-ph]
59. W. Wang, R.L. Zhu, *Eur. Phys. J. C* **75**(8), 360 (2015). doi:[10.1140/epjc/s10052-015-3583-6](#). arXiv:[1501.04493](#) [hep-ph]
60. A. Bussone et al. [ETM Collaboration], *Phys. Rev. D* **93**(11), 114505 (2016). doi:[10.1103/PhysRevD.93.114505](#). arXiv:[1603.04306](#) [hep-lat]
61. R.J. Dowdall et al. [HPQCD Collaboration], *Phys. Rev. Lett.* **110**(22), 222003 (2013). doi:[10.1103/PhysRevLett.110.222003](#). arXiv:[1302.2644](#) [hep-lat]
62. S. Aoki et al., *Eur. Phys. J. C* **74**, 2890 (2014). doi:[10.1140/epjc/s10052-014-2890-7](#). arXiv:[1310.8555](#) [hep-lat]
63. G. Buchalla, A.J. Buras, M.E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996). doi:[10.1103/RevModPhys.68.1125](#). arXiv:[hep-ph/9512380](#)
64. R. Aaij et al. [LHCb Collaboration], *Phys. Rev. Lett.* **111**(18), 181801 (2013). doi:[10.1103/PhysRevLett.111.181801](#). arXiv:[1308.4544](#) [hep-ex]